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Normal vs. Inverted Hierarchy in Type I Seesaw Models

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Abstract

We demonstrate that, for every grand unified model based on a conventional type I seesaw mechanism leading to a normal light neutrino mass hierarchy, one can easily generate a corresponding model with an inverted hierarchy which yields the same neutrino oscillation parameters. However, the latter type model has several unattractive instabilities which will disfavor any grand unified type I seesaw model, if an inverted neutrino mass hierarchy is observed experimentally. This should be contrasted with the softly-broken $L_e - L_\mu - L_\tau$ flavor symmetry models which are eliminated, if the data favors a normal mass hierarchy.

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Since the first convincing evidence in 1998 for atmospheric neutrino oscillations by the Super-Kamiokande (S-K) collaboration and subsequent confirmation of hints of solar neutrino oscillations by both the Sudbury Neutrino Observatory and KamLAND collaborations, great progress has been made in pinning down the values of the leading oscillation parameters Δm^2_{32}, $\sin^2 2\theta_{23}$, Δm^2_{21} and $\sin^2 2\theta_{12}$ [1]. The same can not be said for the other 3-family mixing parameters, $\sin^2 2\theta_{13}$ and the leptonic Dirac and Majorana CP phases δ, χ_1 and χ_2. The angle θ_{13} has been bounded by the CHOOZ and Palo Verde reactor experiments [2] to be noticeably smaller than the other two angles, while the three phases are completely unknown. Moreover, most important for the subject of this letter, even the 3-neutrino mass hierarchy, normal or inverted, remains undetermined. Experimental determination of the hierarchy and θ_{13} will be critical for the continued viability of models in the literature which attempt to explain the neutrino mixing data; cf. [3] for extensive reviews of such models.

For models based on some flavor symmetry, those exhibiting a softly broken $L_e - L_\mu - L_\tau$ lepton number produce only an inverted hierarchy [4]. In contrast, all of the successful grand unified models in the literature employing a conventional type I seesaw mechanism appear to prefer a normal hierarchy; cf. [5] for some still viable models of this type. We seek to explore here why this is the case. With such a dichotomy present, clearly identification of the correct hierarchy observed in Nature will serve to eliminate a large batch of models of one kind or the other. On the other hand, grand unified models based on a type II [6] or type III [7] seesaw with direct or induced Higgs triplet contributions can be constructed by design to have either a normal or an inverted hierarchy.

In this letter we show how, given a satisfactory grand unified model with normal neutrino mass hierarchy based on a type I seesaw, one can easily generate a corresponding model with an inverted hierarchy which also satisfies the same known neutrino oscillation parameters. The issue becomes which hierarchy model is more satisfactory theoretically in terms of flavor symmetry, heavy neutrino mass hierarchy with possibly successful leptogenesis, and stability of the solution. To address this point, we use a much studied $SO(10)$ grand unified model with a $U(1) \times Z_2 \times Z_2$ flavor symmetry proposed by the author in collaboration with S.M. Barr which leads naturally to a normal hierarchy [8]. Results for the light neutrino mass matrices are also derived for the class of models in which the charged lepton mass matrix is diagonal in flavor space. Features obtained in the models considered are more generally expected to hold for the whole class of grand unified models with a type I seesaw mechanism.

We begin with the known neutrino oscillation data and the definitions that apply for the parameters appropriate for a normal vs. inverted hierarchy. Experimentally the atmospheric and solar neutrino mixing parameters are approximately equal to [1,2]

$$\begin{aligned} |\Delta m^2_{32}| &\simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2, & \sin^2 2\theta_{23} &\simeq 1.0, \\ \Delta m^2_{21} &\simeq 7.0 \times 10^{-5}\ \mathrm{ev}^2, & \tan^2 \theta_{12} &\simeq 0.40, \\ \Delta m^2_{31} &\simeq \Delta m^2_{32}, & \sin^2 2\theta_{13} &\lesssim 0.16. \end{aligned} \tag{1}$$

Although the mass squared differences are determined, the actual neutrino mass scale is undetermined as is the mass hierarchy, i.e., whether $\Delta m^2_{32} \equiv m^2_3 - m^2_2$ is positive or negative. Clearly, however, the larger (smaller) mass separation is associated with the atmospheric (solar) neutrino flavor mixing. For the solar neutrinos, the mass squared difference Δm^2_{21} is known to be positive, since the solar mixing angle θ_{12} lies in the first octant.

The Maki-Nakagawa-Sakata (MNS) neutrino mixing matrix with elements $U_{\alpha i}$ relates the flavor state $\alpha = e,\ \mu,\ \tau$ to the mass eigenstates $i = 1,\ 2,\ 3$ in their respective orders. By convention, we have

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2}$$

where the cosines and sines refer to rotations of the mass eigenstates. From the mixing data in Eq. (1), we can approximate $c_{23} \simeq s_{23} \simeq 1/\sqrt{2}$, $s_{12} \simeq \sqrt{2/7}$, $c_{12} \simeq \sqrt{5/7}$, and $s_{13} \lesssim 0.2$. The mass eigenstates are then approximately given in terms of the flavor eigenstates by

$$\begin{aligned} \nu_1 &\simeq \sqrt{\tfrac{5}{7}}c_{13}\nu_e - \sqrt{\tfrac{1}{7}}(\nu_\mu - \nu_\tau) - \sqrt{\tfrac{5}{14}}s_{13}e^{-i\delta}(\nu_\mu + \nu_\tau), \\ \nu_2 &\simeq \sqrt{\tfrac{2}{7}}c_{13}\nu_e + \sqrt{\tfrac{5}{14}}(\nu_\mu - \nu_\tau) - \sqrt{\tfrac{1}{7}}s_{13}e^{-i\delta}(\nu_\mu + \nu_\tau), \\ \nu_3 &\simeq s_{13}e^{i\delta}\nu_e + \sqrt{\tfrac{1}{2}}c_{13}(\nu_\mu + \nu_\tau), \end{aligned} \tag{3}$$

where the states ν_1 and ν_2 are involved with the solar neutrino transitions, while ν_2 and ν_3 are involved with the atmospheric neutrino transitions. As such, the same U_{MNS} mixing matrix leading to the same mass eigenstate flavor compositions applies, whether the hierarchy is normal, $m_1 < m_2 < m_3$, or inverted, $m'_3 < m'_1 < m'_2$, as illustrated in Fig. 1. For the inverted case, $\Delta m^2_{21} = m'^2_2 - m'^2_1$ and $|\Delta m^2_{32}| = m'^2_2 - m'^2_3$ with the mass square differences specified experimentally in Eq. (1).

We now turn to the issue of inverted vs. normal mass hierarchy in grand unified models with the conventional type I seesaw mechanism. Since most, if not all, such models in the literature [5] favor a normal hierarchy, we start with that case. The specified Dirac and right-handed Majorana mass matrices, M_N and M_R, will both have normal hierarchy in that their mass eigenvalues with the smaller values will be more closely spaced. The light left-handed neutrino mass matrix follows from the conventional type I seesaw formula [9] according to

$$M_\nu = -M_N M_R^{-1} M_N^T, \tag{4}$$

which by assumption also has a normal hierarchy, albeit a rather mild one. This mass matrix is diagonalized by the unitary transformation, U_{ν_L}, whereby

$$U^\dagger_{\nu_L} M^\dagger_\nu M_\nu U_{\nu_L} = \mathrm{diag}(m_1^2,\ m_2^2,\ m_3^2). \tag{5}$$

Since the M_ν matrix is complex symmetric, one can choose phases for U_{ν_L} such that diagonalization of M_ν itself occurs with

$$U^T_{\nu_L} M_\nu U_{\nu_L} = \mathrm{diag}(m_1,\ m_2,\ m_3), \tag{6}$$

where the mass eigenvalues are real and positive with $m_1 < m_2 < m_3$. The MNS mixing matrix is given by the product of the two unitary matrices, U_{L_L} which diagonalizes the charged lepton mass matrix, and U_{ν_L} above according to

$$\begin{aligned} V_{MNS} &\equiv U^{\dagger}_{L_L} U_{\nu_L} = U_{MNS}\Phi, \\ \Phi &= \mathrm{diag}(e^{i\chi_1},\ e^{i\chi_2},\ 1), \end{aligned} \tag{7}$$

where U_{MNS} is specified by the convention of Eq. (2), and Φ is a diagonal phase matrix involving the two Majorana phases, χ_1 and χ_2.

Note that if the right-handed Majorana mass matrix had the structure $M_R \propto M_N^T M_N$, by the seesaw formula in Eq. (4), M_{ν_L} would be proportional to the identity matrix with three-fold degeneracy. This suggests that some satisfactory M'_R may be found leading to an inverted neutrino mass hierarchy. We now determine the corresponding matrices which yield not only the correct mass spacing with an inverted hierarchy, but also the correct MNS mixing matrix.

For this purpose, let us choose the heaviest light neutrino mass m'_2 to be assigned some value to be specified later. The other light neutrino masses can then be determined from the observed mass squared differences in Eq. (1) and m'_2 according to

$$m'_1 = \sqrt{m'^2_2 - \Delta m^2_{21}}, \quad m'_3 = \sqrt{m'^2_2 - |\Delta m^2_{32}|}. \tag{8}$$

We invert the counterpart of Eq. (6) written for the inverted hierarchy case and identify the resultant M'_ν with the seesaw formula involving the same Dirac neutrino matrix but the new right-handed Majorana matrix, M'_R:

$$\begin{aligned} M'_\nu &= U^*_{\nu_L} \mathrm{diag}(m'_1,\ m'_2,\ m'_3) U^{\dagger}_{\nu_L} \\ &= -M_N M'^{-1}_R M_N^T. \end{aligned} \tag{9}$$

By using the same unitary tranformation U_{ν_L} for the inverted hierarchy case as in the normal hierarchy case and the masses m'_i with $m'_3 < m'_1 < m'_2$ ordered in the diagonal matrix as above, we guarantee that the same desired MNS mixing matrix, U_{MNS}, and Majorana phase matrix, Φ, are obtained in the inverted case. In general one will find that the mild normal hierarchy for the light neutrino mass matrix, M_ν, is replaced by a completely different texture with no simple flavor symmetry obvious for the inverted hierarchy version. The new right-handed Majorana matrix which accomplishes this follows from the seesaw formula in Eq. (9) by inversion:

$$M'_R = -M_N^T M'^{-1}_\nu M_N. \tag{10}$$

The procedure described above for determining the new light neutrino mass matrix M'_ν which obtains in the inverted hierarchy case, given the original normal hierarchy model, can be applied for any grand unified model with a type I seesaw mechanism. It ensures that the neutrino oscillation data for the mixing matrix will again be satisfactorily fit but now with an inverted neutrino mass spectrum. Of crucial importance is the fact that the Dirac neutrino mass matrix is the same for both hierarchies, for with a family unification group such as $SO(10)$, this mass matrix is closely related to that for the up quark sector, since the same set of Higgs representations are involved. This means that the two different hierarchies arise solely from the different right-handed Majorana mass matrix textures. Hence the heavy right-handed neutrino spectra will differ, as will the possibilities for successful leptogenesis.

The nature of the light neutrino spectrum of course has a direct bearing on the possible observation of neutrino-less double beta decay. The relevant parameter is the effective mass which can be written as

$$\langle m_{ee} \rangle = |\sum_j m_j \, (U_{MNS}\Phi)^2_{1j} \,|. \tag{11}$$

The same relation applies for both the normal and inverted hierarchies, with m_j for the normal hierarchy replaced by the primed counterparts, m'_j, in the inverted case, with $j = 1, 2, 3$ taken in the same order.

To make the distinctions more obvious, we shall first use as an example a much studied $SO(10)$ grand unified model which explains well the quark mass and mixing data, as well as the lepton mass and mixing data with a natural normal neutrino mass hierarchy [8]. Based on the $SO(10)$ family symmetry with a $U(1) \times Z_2 \times Z_2$ flavor symmetry, one adjoint $\mathbf{45}_H$, two pairs of $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$, along with several Higgs fields in the $\mathbf{10}_H$ and singlet representations, the nine quark and charged lepton masses and four CKM quark mixing angles and CP phase are well fitted with eight model parameters. The lopsided nature of the down quark and charged lepton mass matrices, arising from an electroweak symmetry-breaking Higgs field in the $\mathbf{16}_H$ representation, readily explains the small V_{cb} quark mixing and near maximal $U_{\mu 3}$ atmospheric neutrino mixing for any reasonable right-handed Majorana neutrino mass matrix, M_R [10]. The type of solar neutrino solution is found to be controlled mainly by M_R in this model. In fact, the uncertain nature of this M_R matrix has closely paralleled the uncertainty in the neutrino mixing data mentioned in the introduction. When the model was initially proposed, a very simple form of M_R led directly to the small mixing angle (SMA) solar neutrino solution favored by experiment at that time. A more complicated structure with zero subdeterminant for the $2-3$ sector was later realized to obtain the large mixing angle (LMA) solar neutrino solution [11]. The near degeneracy of the two lightest right-handed Majorana neutrinos and the resulting possibility of successful resonant leptogenesis was recognized in [12].

We shall simply begin with the relevant mass matrices and refer the interested reader to the literature cited for more specific details of this model. The two Dirac mass matrices for the charged leptons and neutrinos are given by

$$M_N = \begin{pmatrix} \eta & \delta_N & \delta'_N \\ \delta_N & 0 & \epsilon \\ \delta'_N & -\epsilon & 1 \end{pmatrix} m_U, \quad M_L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon \\ \delta' e^{i\phi} & -\epsilon & 1 \end{pmatrix} m_D. \tag{12}$$

Here the Dirac matrices are written with the convention that the left-handed fields label the rows and the left-handed conjugate fields label the columns. All nine quark and charged lepton masses, plus the three CKM angles and CP phase, are well fitted with the eight input parameters,

$$\begin{aligned} m_U &\simeq 113 \text{ GeV}, & m_D &\simeq 1 \text{ GeV}, \\ \sigma &= 1.83, & \epsilon &= 0.147, \\ \delta &= 0.00946, & \delta' &= 0.00827, \\ \phi &= 119.4^\circ, & \eta &= 6 \times 10^{-6}, \end{aligned} \tag{13}$$

defined at the GUT scale to fit the low scale observables after evolution downward from Λ_{GUT}. As initially proposed, the two parameters δ_N and δ'_N were set equal to zero, but we shall allow them to assume also non-zero, but very small, values which can lead to a more nearly satisfactory result for leptogenesis [13]. Such values considered are small enough so as not to destroy the good fit with experiment obtained initially.

The most general form for the right-handed Majorana mass matrix considered in [11] which gives the large mixing angle (LMA) solar neutrino solution is

$$M_R = \begin{pmatrix} c^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R, \tag{14}$$

as explained by the structure of the Froggatt-Nielsen diagrams [14]. The same parameter, ϵ, appears as in Eqs. (12) and (13) under the assumption of a universal coupling of the $\mathbf{45}_H$ Higgs field to all the matter fields. For the parameters originally chosen in the model, this matrix exhibits a normal hierarchy which departs from a simple geometrical form when not all three of $a,\ b$ and c are equal. For the simple case where $\delta_N = \delta'_N = 0$, the seesaw formula of Eq. (4) determines the light neutrino mass matrix to be

$$\begin{aligned} M_\nu &= -M_N M_R^{-1} M_N^T \\ &= -\begin{pmatrix} 0 & \frac{1}{a-b}\epsilon & 0 \\ \frac{1}{a-b}\epsilon & \frac{b^2-c^2}{(a-b)^2}\epsilon^2 & \frac{b}{b-a}\epsilon \\ 0 & \frac{b}{b-a}\epsilon & 1 \end{pmatrix} m_U^2/\Lambda_R. \end{aligned} \tag{15}$$

The rather extreme hierarchy of M_R is more than balanced by the normal hierarchy of M_N and M_N^T to yield a rather mild normal hierarchy for M_ν for selected values of $a,\ b$ and c.

We proceed to illustrate the connection between the inverted and normal versions of the model with a numerical example. Let us choose the following parameters which lead to a normal hierarchy,

$$\begin{aligned} &a = 0.25 + 0.15i, && b = 1.20 + 0.90i, \\ &c = 0.25 + 0.25i, && \Lambda_R = 2.90 \times 10^{14}\ \text{GeV}, \\ &\delta_N = -0.65 \times 10^{-5}, && \delta'_N = -1.0 \times 10^{-5}, \end{aligned} \tag{16}$$

and were recently found to yield nearly successful baryogenesis with a resonant leptogenesis mechanism [13].

With this choice of parameters listed, the right-handed Majorana matrix is given by

$$\begin{aligned} M_R = &\begin{pmatrix} 4.5i \times 10^{-12} & -(1.058 + 0.794i) \times 10^{-6} & (1.5 + 0.9i) \times 10^{-6} \\ -(1.058 + 0.794i) \times 10^{-6} & 0.0216 & -0.147 \\ (1.5 + 0.9i) \times 10^{-6} & -0.147 & 1 \end{pmatrix} \\ &\times 2.9 \times 10^{14}\ \text{GeV}, \end{aligned} \tag{17}$$

from which the seesaw formula of Eq. (4) yields

$$M_\nu = -\begin{pmatrix} (7.03-5.55i)\times 10^{-5} & -0.0954+0.0753i & -(8.16-4.79i)\times 10^{-5} \\ -0.0954+0.0753i & 0.238-0.165i & 0.341-0.130i \\ -(8.16-4.79i)\times 10^{-5} & 0.341-0.130i & 1 \end{pmatrix} \times 0.0440 \text{ eV}. \tag{18}$$

Some results obtained in the model for this normal hierarchy version are

$$\begin{aligned} & M_3 = 2.96\times 10^{14},\ M_2 \simeq M_1 = 3.06\times 10^{8} \text{ GeV}, \\ & m_3 = 51.0, \quad m_2 = 8.8, \quad m_1 = 2.8 \text{ meV}, \\ & \langle m_{ee}\rangle = 0.44 \text{ meV}, \quad \eta_B = 2.2\times 10^{-10}, \end{aligned} \tag{19}$$

for the heavy right-handed and light left-handed neutrino masses, the effective neutrino-less double beta decay mass $\langle m_{ee}\rangle$, and the net baryon number to photon number asymmetry ratio for the Universe, η_B. The very small departures of δ_N and δ'_N from zero serve to make the lighter two heavy right-handed neutrinos more nearly degenerate and thus enhance the resonant leptogenesis effect. We refer the reader to reference [12] for the explicit neutrino mixing angle results which are in excellent agreement with the present data.

To obtain a corresponding model with inverse hierarchy, we select $m'_2 = 0.10$ eV, which lies below the maximum of 0.23 eV allowed @ 95% c.l. for nearly degenerate neutrinos by the recent WMAP data in combination with the 2dF Galaxy Redshift Survey and Lyman-α forest power spectrum [15]. The procedure elaborated above in Eqs. (8) and (9) with the help of the unitary transformation matrix,

$$U_{\nu_L} = \begin{pmatrix} 0.799+0.353i & -0.198+0.443i & -0.0134+0.0391i \\ 0.446+0.115i & 0.196-0.772i & 0.130-0.370i \\ -0.157+0.020i & 0.028+0.360i & 0.002-0.918i \end{pmatrix} \tag{20}$$

determined from the normal hierarchy case as in Eq. (6), then leads to the light neutrino mass matrix,

$$M'_\nu = -\begin{pmatrix} -0.416+0.467i & -0.744+0.604i & 0.317-0.109i \\ -0.744+0.604i & 0.565-0.346i & 0.098-0.078i \\ 0.317-0.109i & 0.098-0.078i & 1 \end{pmatrix} \times 0.0836 \text{ eV}, \tag{21}$$

in the inverted hierarchy version considered, from which the required M'_R is determined to be

$$M'_R = \begin{pmatrix} (1.397+0.318i)\times 10^{-10} & (1.340-0.025i)\times 10^{-6} & -(1.028+0.058i)\times 10^{-5} \\ (1.340-0.025i)\times 10^{-6} & 0.0204-0.0006i & -0.142+0.003i \\ -(1.028+0.058i)\times 10^{-5} & -0.142+0.003i & 1 \end{pmatrix} \times 1.46\times 10^{14} \text{ GeV}. \tag{22}$$

From Eq. (8) and these matrices we find

$$\begin{aligned}
&m'_2 = 0.1000,\ m'_1 = 0.0996,\ m'_3 = 0.0866 \text{ eV}, \\
&M'_3 = 1.49 \times 10^{14}, \quad M'_2 = 3.10 \times 10^{10},\ M'_1 = 8.77 \times 10^{3} \text{ GeV}, \\
&\langle m'_{ee} \rangle = 0.044 \text{ eV}, \quad \eta'_B = 8.5 \times 10^{-22}.
\end{aligned} \tag{23}$$

While M_R and M'_R do not appear to be significantly different, the opposite signs of the 12, 21 and 13, 31 elements for the two matrices play an important role. Moreover, the 2-3 sector subdeterminant fails to vanish in the latter case. This accounts for the huge hierarchy in the right-handed neutrinos in the inverted case, which departs from the near degeneracy of the two lighter right-handed neutrinos in the normal case. As a result, the nearly successful resonant leptogenesis solution with the normal hierarchy is completely lost. The heavy right-handed neutrinos in the inverted case span such a huge hierarchy that the lightest is much lighter than the $10^9 - 10^{10}$ GeV required for successful leptogenesis without a resonant enhancement [16].

More striking are the differences in the light left-handed neutrino sector, where the mass spectrum is indeed inverted but nearly degenerate by the choice of m'_2, as illustrated in Fig. 1. While the normal hierarchy M_ν matrix has two near texture zeros in the 11, 13 and 31 elements, the elements of M'_ν are all rather comparable in magnitude. This is arises since the determinant of M'_ν is three orders of magnitude larger than that for M_ν, i.e., $|m'_3 m'_2 m'_1| \sim 700 |m_3 m_2 m_1|$. Such an inverted spectrum is highly unstable against very small changes in the matrix elements of M'_R. It is also very unstable against radiative corrections upon evolution downward from the GUT scale, since two of the nearly degenerate masses have approximately the same CP parity [17]. Mainly for these reasons, we consider the inverted hierarchy solution in this model to be much less satisfactory than the normal hierarchy solution.

We regard this result to be a general feature of grand unified models with a type I seesaw mechanism. Unfortunately there are very few models in the literature as predictive as this with which we are prepared to test this prediction. However, we can further illustrate the claim by considering the whole class of models for which the charged lepton mass matrix is diagonal in flavor space. Recall that for the specific model examined in detail above, the charged lepton mass matrix is lopsided in that basis.

With the charged lepton mass matrix, M_L, diagonal in the flavor basis, the unitary transformation, U_{L_L}, is just the identity matrix. Successful models in this class then require that $U_{\nu_L} \simeq U_{MNS}$ by Eq. (7), where for our purposes we can drop the two Majorana phases without loss of generality. We set U_{MNS} equal to that in Eq. (2) with the approximations $c_{23} \simeq s_{23} \simeq 1/\sqrt{2}$, $s_{12} \simeq \sqrt{2/7}$, $c_{12} \simeq \sqrt{5/7}$ and $s_{13} = 0$. By selecting the normal hierarchy neutrino masses found earlier, and inverting Eq. (6) we find for the light neutrino mass matrix,

$$\begin{aligned}
M_\nu &= U^*_{\nu_L} \text{diag}(0.0028,\ 0.0088,\ 0.051) U^\dagger_{\nu_L} \\
&= \begin{pmatrix} 0.451 & 0.192 & -0.192 \\ 0.192 & 2.904 & 2.196 \\ -0.192 & 2.196 & 2.904 \end{pmatrix} \times 0.01 \text{ eV}.
\end{aligned} \tag{24}$$

This matrix clearly has the texture for a reasonably stable normal mass hierarchy.

To investigate a corresponding inverted mass hierarchy version, we again select $m'_2 = 0.10$ eV and find with the same $U_{\nu_L} = U_{MNS}$ the new light neutrino mass matrix,

$$\begin{aligned} M'_\nu &= U^*_{\nu_L}\text{diag}(0.0996,\ 0.10,\ 0.0866)U^\dagger_{\nu_L} \\ &= \begin{pmatrix} 9.975 & 0.011 & -0.011 \\ 0.011 & 9.325 & -0.665 \\ -0.011 & -0.665 & 9.325 \end{pmatrix} \times 0.01 \text{ eV}. \end{aligned} \tag{25}$$

Note that if the small off-diagonal elements of this matrix were neglected, the neutrino mass hierarchy would be normal with the lowest two levels degenerate. It is only the small 23 and 32 off-diagonal corrections which convert the hierarchy into an inverted one. This matrix is thus much more sensitive to small changes in the underlying right-handed Majorana matrix structure for each model in the class considered. On the other hand, the dominant diagonal elements set the scale for the determinant of the matrix and thus the product of the three neutrino masses, which again is about 700 times larger than that for M_ν.

These results should be contrasted with those for models involving a softly broken $L_e - L_\mu - L_\tau$ flavor symmetry. There the light neutrino mass matrix has large 12, 13, 21 and 31 matrix elements with small values for the other elements. Hence the results are less sensitive to instabilities resulting from small perturbations away from the desired entries, and in such models no evolution from a high mass scale is typically involved.

In summary, we have shown that to every successful normal neutrino mass hierarchy solution of a grand unified model corresponds an inverted hierarchy solution with exactly the same MNS mixing matrix. The inverted left-handed neutrino spectrum illustrated is nearly degenerate and is highly unstable to small changes in the parameters for the right-handed Majorana mass matrix as well as to radiative corrections upon evolution from the grand unified scale. This suggests that future observation of an inverted hierarchy would tend to disfavor grand unified models based on the conventional type I seesaw mechanism. On the other hand, grand unified models with type II or type III seesaw mechanisms, where a left-handed Majorana mass matrix can arise from direct or induced Higgs triplet contributions, or models based on a conserved $L_e - L_\mu - L_\tau$ lepton number would then be favored. Conversely, observation of a normal hierarchy would eliminate the conserved $L_e - L_\mu - L_\tau$ number models in favor of the grand unified models. Successful leptogenesis in the latter type models is also more favorable.

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FIGURES

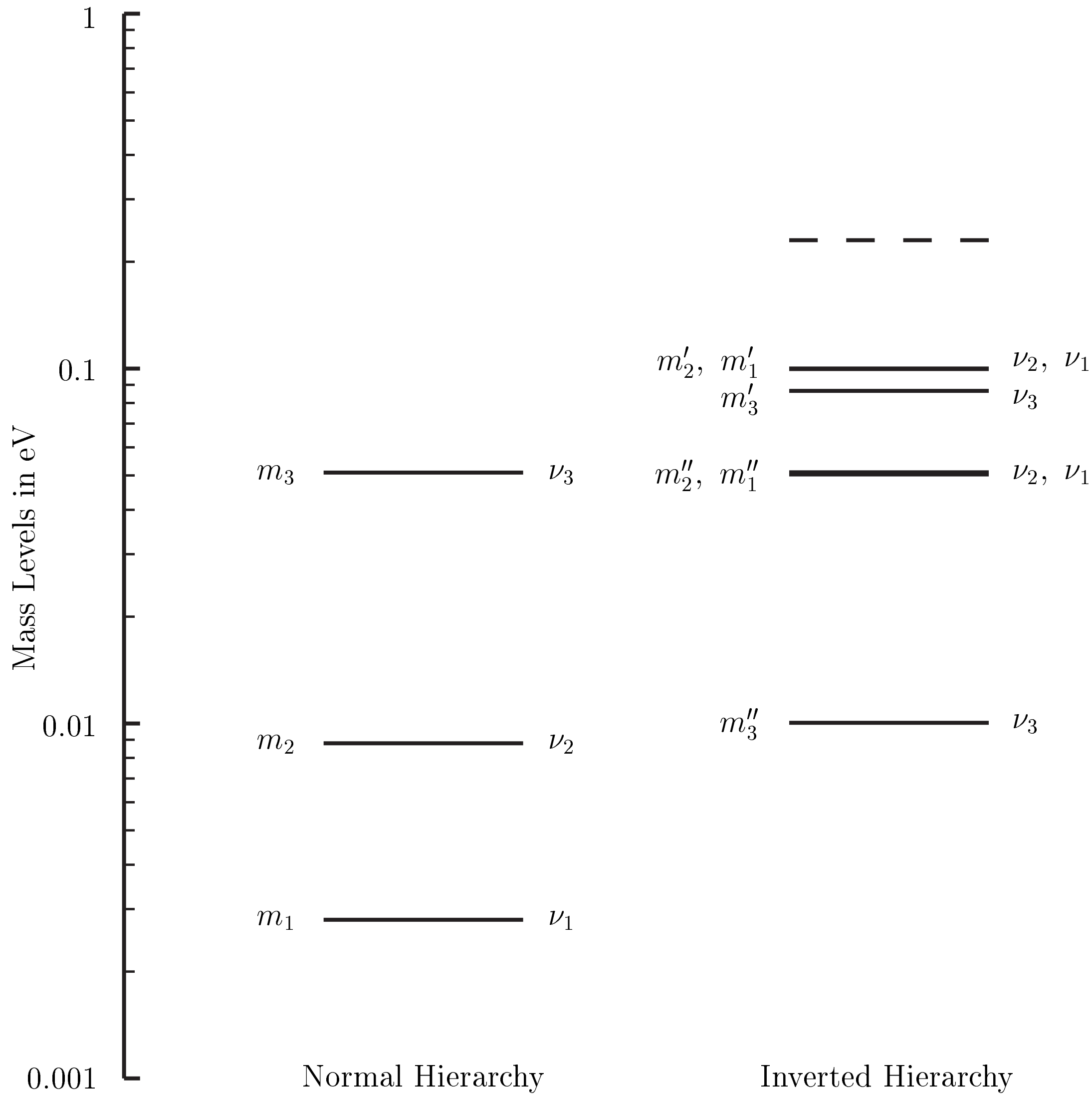

Figure 1: Normal hierarchy and two examples of inverted hierarchy. The mass eigenstates labeled by ν_1, ν_2, ν_3 have the flavor content indicated in Eq. (3). The states ν_1 and ν_2 for the inverted hierarchy are so close in mass that these states appear to overlap for the mass scale chosen. The dashed line represents the largest nearly degenerate levels allowed by the combination of WMAP, 2dF Galaxy Redshift Survey, and Lyman-α data.